



Public Debt Sustainability in Resource-Rich Countries with Access to Capital Markets: The Case of Peru

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Abstract. This paper examines the relevance of economic growth, currency depreciation, cost of public financing, inflation, and export prices on the sustainability of Peru's public debt. The measurements correspond to different specifications of vector autoregressive (VAR) models applied to macro-fiscal data from 1999-2018. The results validate the relevance of all the factors for the short, medium, and long term. Furthermore, these results suggest that future macro-fiscal policy measures should aim to continue actions to improve the public debt profile and diversify fiscal revenues, in order to reduce the vulnerability of fiscal sustainability due to unfavorable changes in export prices.

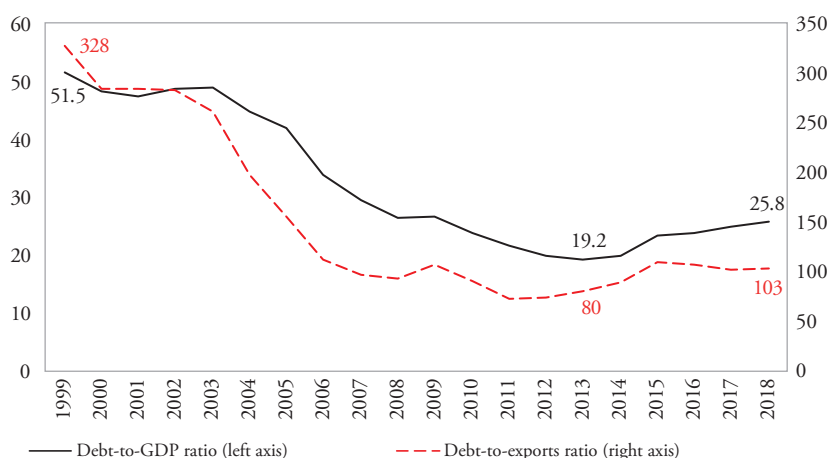
Keywords: sustainability, debt, economic growth, currency depreciation, export prices.

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Introduction

After the financial crisis of 2007-2008, the Peruvian economy's public debt-to-GDP ratio was among the lowest in the world, below that of its Latin American peers Colombia, Mexico, and Brazil (Dobbs et al., 2015). Over the twenty-year period spanning the start of 1999 to the end of 2018, Peru's debt-to GDP ratio was halved (from 51.5% to 25.8%), while its public debt-to-export ratio fell to a third of the initial total (from 328% to 103%)² (see Figure 1). However, when the time window is reduced to the final six years, it can be seen that the public debt ratio rose from 19.2% at the start of 2013 to 25.8% by the end of 2018.

Figure 1
Evolution of public debt (%)



Source: compiled by author based on Banco Central de Reserva del Perú (BCRP) data.

Given this context, the present study seeks to answer the following questions: What factors lay behind the sustained reduction in Peru's public debt ratio from 1999 to 2018? In what way did these factors contribute during this period? What challenges and opportunities do these factors pose for macro-fiscal policy in the medium and short term?

To deal with these very questions, in 2002, the IMF designed a Debt Sustainability Analysis Framework for Market-Access Countries (MAC DSA)

2 The most common ratios used in the evaluation of debt sustainability are those that compare total debt (solvency indicators, which focus on the total or long-term commitment) or debt service (liquidity indicators, which address immediate or short-term obligations) with exports or GDP. However, the International Monetary Fund has developed additional ratios with the aim of analyzing other debt characteristics besides solvency and liquidity. (See <http://www.imf.org>)

(FMI, 2002).³ This framework links the public debt dynamics of economies that have significant and sustainable access to international capital markets (that is, all advanced economies and most emerging ones) with factors that capture conditions of economic growth, exchange rate depreciation and the real cost of public borrowing. However, this framework is insufficient for analysis of the Peruvian case, because Peru is not only an emerging, middle-income country with access to international capital markets but is also rich in national resources, which makes it highly dependent on the terms of trade. Therefore, this article will build on the MAC DSA by incorporating a direct link between export prices and the primary balance.

Use of the MAC DSA is considered consistent with a focus on export-price dependency for the Peruvian economy, in that: (i) real GDP growth is a key indicator of the ability of income to service debt payments; (ii) better financing conditions, linked also to the proportion of public debt at fixed rate (see Figure 2), limit the pace at which the debt stock can grow; (iii) exchange rate depreciation represents a risk, given the considerable degree of dollarization of the debt stock (see Figure 2); and (iv) Peru is a country rich in natural resources⁴ and, as such, a rise in the price of export commodities allows for greater fiscal revenues,⁵ largely in the form of income tax paid by mining companies, levies, and royalties.

The aim of this study is to quantify the implications that the conditions of economic growth, exchange rate depreciation, cost of public borrowing, and export prices have on public debt sustainability in a country rich in natural resources with access to international capital markets.

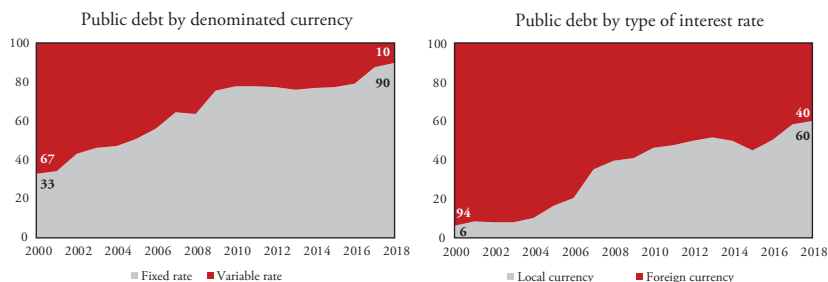
Similarly, the magnitudes, significance, and duration of the effects that these factors have on fiscal sustainability also imply opportunities and, to a greater extent, risks for economic policy.

3 Since 2002, the MAC DSA has undergone many revisions aimed at strengthening its analysis. In addition, the World Bank (2005) later added a Debt Sustainability Analysis Framework for Low Income Countries (LIC DSA).

4 According to the US Geological Survey (2019), in 2018, Peru was the second-largest global producer of zinc, copper, and silver; and the sixth-largest of gold.

5 Between 1999 and 2018, government tax revenues have displayed a high level of correlation with the export price index (0.88).

Figure 2
Profile of public debt (%)



Source: compiled by author based on BCRP and Ministry of Education and Finance (MEF) data.

This article is divided into six sections. After this introduction, the second section presents the main concepts related to public debt sustainability, as well as various prior studies centered on the analysis of fiscal sustainability. The third section analyzes the macro-fiscal variables employed, and the possible relationships between them. The fourth section presents the methodological strategy proposed to capture the effects of interest. The fifth section examines how public debt reacts to shocks to the macro-fiscal variables, and to public debt ratio projections under different scenarios. Finally, the sixth section presents the conclusions from the evaluation of Peru's debt sustainability over the period 1999–2018.

1. Theoretical framework

Conceptual framework

Public debt sustainability can be defined by way of various approaches. An initial approach focuses on a context in which fiscal policy does not require significant future corrections; that is, one in which there are no complicated obstacles to meeting financial obligations at a given moment and under the terms originally agreed upon. Failing this, public debt will be sustainable if, with certain speed and efficiency, it is possible to implement budgetary measures that sustain the level of debt (Pereyra, 2003); of course, this is not to imply that such a scenario translates into reality. From a technical perspective, sustainability stems from compliance with an inter-temporal budget constraint in which debt is not accumulated excessively and can be offset by expected future primary surpluses of equal present value (Luporini, 2000). Finally, from a pragmatic point of view, it is inferred that debt will be sustainable if debt ratios follow a future downward trajectory or stabilize at low levels (Herrera & Mendoza,

2004). For this to be so, debt must not rise more rapidly than income or the ability to repay.

Moreover, the assumption of sustainable public debt implies that the government is solvent, a characteristic that is subject to the confidence of creditors and the credibility of authorities (Bachellerie & Couillault, 2005). The chief link between solvency and sustainability lies in the aptitude of a government to discharge its future or long-term financial obligations under the terms originally agreed upon with the creditor, in order to prevent explosive growth of the debt (Pereyra, 2003). The main point of departure between the two concepts is that solvency is a theoretical term with practical limitations, since a government can be solvent in one period and cease to be so in the next (Bachellerie & Couillault, 2005). In turn, sustainability makes it possible to contextualize the dynamics of debt over time, and so expressing it in these terms is preferable.

Background

Research with a similar focus as the present study, but centered on the Colombian case, include Arango and Posada (2001), Díaz and Gutiérrez (2005), and Galvis, Quintero and Rhenals (2008), which break down public debt dynamics into their main determinants, such as primary balance, internal and external interest rates, exchange rate depreciation, and product growth. For Peru, Herrera and Mendoza (2004) devise a theoretical model of public debt sustainability for a small, open economy with free movement of capital, and find that in a context in which there are constraints on raising taxes or reducing public spending, the public debt ratio is pushed up by a reduction in the real GDP growth rate, an increase in currency devaluation, or an increase in the external interest rate.

Conversely, when there is greater freedom to formulate fiscal policy, an improvement in the fiscal balance will suffice to keep the debt ratio stable (Herrera & Mendoza, 2004).

With regard to economic growth and the cost of servicing public debt, Qin et al. (2005) and Ncube and Brixiová (2015) argue that debt sustainability depends on the primary balance, and that this, in turn, depends on the interest rate–growth differential (IRGD). Qin et al. (2005) note that debt sustainability has a feedback relationship between economic growth and interest rates. For their part, Ncube and Brixiová (2015) detect that the IRGD is the main driver of debt sustainability in several African countries, which highlights the importance of promoting dynamic economic activity and making efficient use of the borrowing space in public spending. These results are intuitive, in that an upturn in economic efficiency reduces the

need for external financing and enhances the prospects of economic growth by way of investment; meanwhile, a decrease in interest rates helps to service debt and improves the country risk perception, which ultimately translates into even lower interest rates (Pereyra, 2003).

On the other hand, academic approaches geared toward countries rich in natural resources can be found in Córdova and Rojas (2010), Lanteri (2015), Ganiko, Melgarejo and Montoro (2016), and Ganiko and Montoro (2017). First, Lanteri (2015) points out that the notable recovery of the terms of trade of Latin American economies enabled positive results in their fiscal accounts for 2014. Moreover, the author concludes that there are long-term relationships between the current accounts, primary balance, investment, and terms of trade (Lanteri, 2015). For Peru, Córdova and Rojas (2010), and Ganiko and Montoro (2017) propose various fiscal policy options conditional upon the behavior of the terms of trade. Finally, Ganiko et al. (2016) estimate the fiscal space in emerging economies, stressing the implications of commodity prices for the primary balance. As far as methodological antecedents are concerned, examples include Celasun, Debrun and Ostry (2006), Tanner and Samake (2008), and Anaya and Pienkowski (2015). First, drawing on data from emerging economies, Celasun et al. (2006) implement a vector autoregression (VAR) model for determinants of public debt dynamics (domestic and foreign debt interest rates, economic growth rates, and real exchange rate).

They also use panel data methods to estimate a fiscal reaction function (FRF) and establish connections between the primary balance and its determinants, such as oil prices. Then, they employ the results of the VAR model and the FRF in the debt dynamics equation to develop stochastic projection tools known as fan charts. The methodological strategy developed by Celasun et al. (2006) was used by the IMF in the evaluation of the fiscal sustainability of various countries.⁶

For their part, Tanner and Samake (2008) analyze public debt sustainability in Brazil, Mexico, and Turkey, applying a VAR model to the determinants of debt dynamics (primary deficit, exchange rate, public debt interest rate, and an indicator of industrial production), and add oil prices. Finally, Anaya and Pienkowski (2015) employ a structural VAR (SVAR) model for the determinants of debt dynamics (primary balance, economic growth

⁶ According to the World Bank and the IMF (2012), the strategy developed by Celasun et al. (2006) was implemented, in some cases with slight modifications, in South Africa (2005), Morocco (2008), Mauritania (2010), El Salvador (2010), Indonesia (2010), Israel (2011), and Costa Rica (2011).

rates, inflation rate, and public debt interest rate). Similarly to Celasun et al. (2006), Tanner and Samake (2008) and Anaya and Pienkowski (2015) utilize the results from their respective autoregressive models in the public debt dynamics equation. On this point, Anaya and Pienkowski (2015) develop impulse response functions for the public debt ratio.

Given that the aim of the present study is to establish and study channels between public debt dynamics and export prices, it is worthwhile to review the literature related to the FRF.

An FRF relates an indicator of fiscal effort or management with factors that may be influencing it. Much of the literature stresses that the variable that best reflects the effort or behavior of the fiscal policymaker is the primary balance;⁷ however, there are also studies that propose fiscal revenues or a specific form of tax income,⁸ fiscal expenditure,⁹ or public debt.¹⁰

The literature expresses these variables in a range of ways: in terms of level, percentage of GDP, cyclical adjustment, differences, or percentage variation.

Nor is there consensus on which of the variables are explanatory in the specification of an FRF. For instance, many studies point to lags of the primary balance and public debt. Others require the presence of an indicator of economic activity, such as economic growth rate,¹¹ output gap,¹² among others.¹³ However, thus far few studies have incorporated variables related to commodities.¹⁴ Finally, the literature contains some analyses that apply controls for policy factors, and for fiscal rules and advice.¹⁵

2. Analytical Framework

Data

The data employed in the present study are of quarterly frequency and span the 1999–2018 time horizon. This period was selected because: (i) there is no available data for at least one of the necessary series corresponding to earlier periods; and (ii) even if this data were available, it would not be

7 See Celasun et al. (2006); Golinelli and Momigliano (2007); De Mello (2008), Afonso and Jalles (2011); and Plödt and Reicher (2015).

8 See Dore and Masson (2002); Afonso (2005); De Mello (2008); and Galvis (2015).

9 See Dore and Masson (2002); Afonso (2005); and Burger and Marinkov (2012).

10 See Afonso and Jalles (2011).

11 See Adedeji and Williams (2007).

12 See Celasun et al. (2006); Afonso and Jalles (2011); De Mello (2008); and Plödt and Reicher (2015).

13 Véanse Adedeji and Williams (2007); and El Mahmah and Kandil (2019).

14 See Celasun et al. (2006); and Checherita-Westphal and Ždarek (2015).

15 See Adedeji and Williams (2007); De Mello (2008); and Checherita-Westphal and Ždarek (2015).

prudent to include it given the instability of the relevant variables in the 1980s and 1990s. Moreover, it should be noted that the series of effective nominal public debt interest rates, including those denominated in local and foreign currency, are inferred based on the interest paid as a fraction of the public debt stock.¹⁶

Relationship between variables

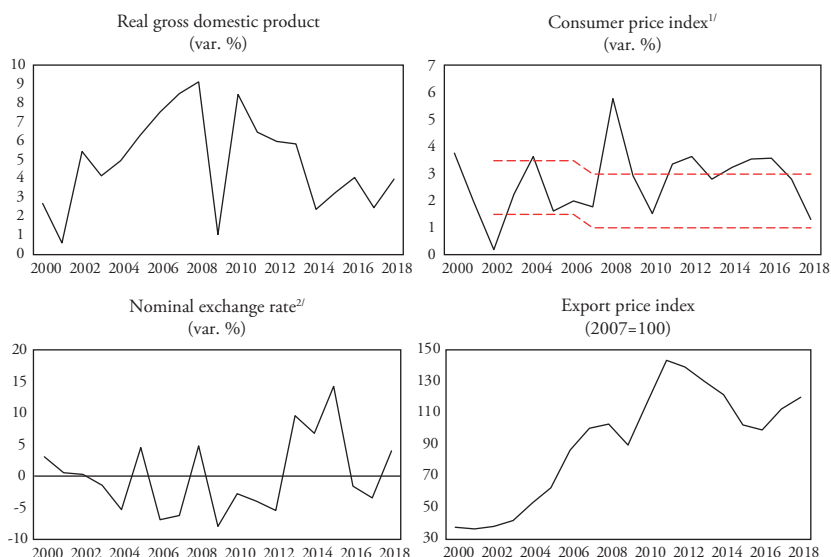
This section provides an overview of the variables to be employed. In addition, it explores the possible relationships between the variables by way of a cyclic cross-correlation analysis. Finally, to determine the econometric process to be followed, it examines series stationarity and possible causal relationships.

a. Evolution of variables

Figure 3 shows the evolution of the macroeconomic variables of interest: variations in real GDP, the consumer price index, the nominal exchange rate, and the export price index. In turn, Figure 4 shows the evolution in the fiscal accounts that are of interest for this study: the determinants of primary balance and public-debt stock and the respective spending on interest.

¹⁶ This is based on equation (2), presented in Appendix 1.

Figure 3
Macroeconomic variables



Notes

^{1/} The red lines represent the target inflation range of the Peruvian Central Bank (Banco Central de Reserva del Perú, BCRP), which was 1.5%–3.5% between 2002 and 2006, and 1%–3% starting in 2007.

^{2/} For reasons of data availability, between 1999 and 2002, this corresponds to evolution in the average exchange rate over the period; and for 2003 onward it corresponds to evolution in the end-of-period exchange rate.

Source: compiled by author based on BCRP data.

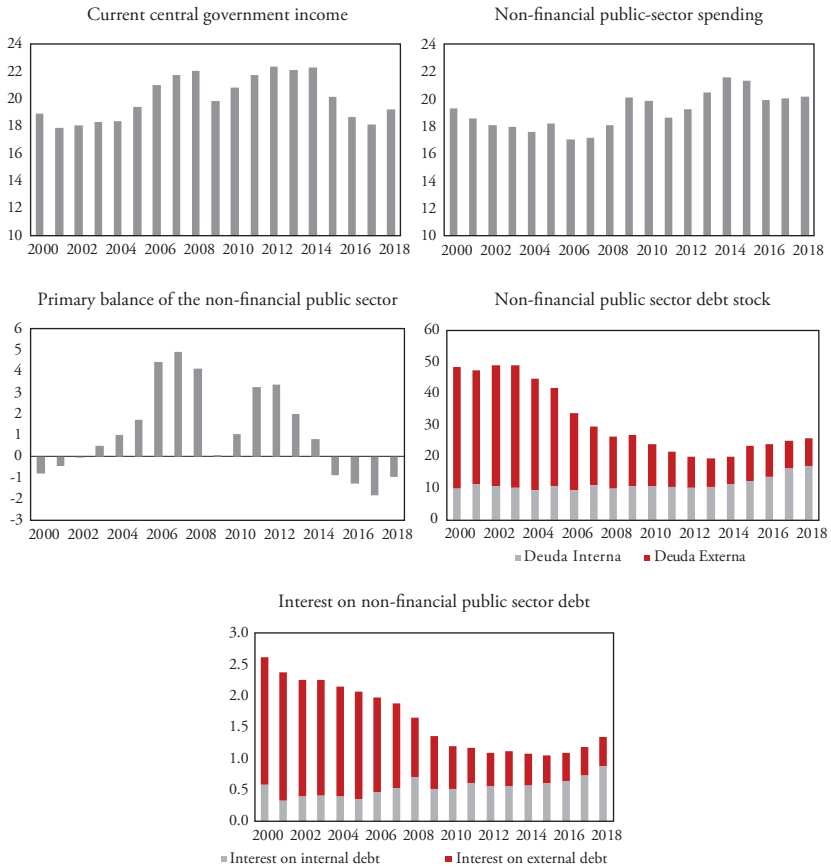
By the start of the 2000s, most of the economic measures adopted during the 1990s¹⁷ had matured. This—reflected in an ordering and consolidation of the main economic accounts and in a sustained increase in the international prices of the major export commodities—allowed the economy to grow at ever-increasing rates while inflation and currency depreciation remained at relatively low levels. Thus, during the period 2000–2008, the economy grew by an average of 2.6%, with an average exchange rate fluctuation of -0.7%.

Over the same period, outstanding management of public ensured fiscal sustainability. On the one hand, the primary balance went from a deficit

17 The measures were related, to among other things, greater openness to trade; promotion of private entrepreneurial activity; and better management of monetary aggregates, financial aggregates (in 1992 the Organic Law for the Central Reserve Bank of Peru was passed, providing for the autonomy of that entity; and in 2002 an inflation targeting system was adopted), and fiscal aggregates (in 1999 the Fiscal Responsibility and Transparency Law was promulgated and the first fiscal rules implemented; and in 2000 the first Multiannual Macroeconomic Framework was published).

of 0.8% in 2000 to a surplus of 4.2% in 2008, as a result of nominally greater income. On the other hand, the public debt stock as a percentage of GDP fell significantly, from 48.3% in 2000 to 26.4% in 2008. Finally, the interest paid as a percentage of GDP also dropped considerably, from 2.6% in 2000 to 1.7% in 2008, providing leeway for greater productive expenditure.

Figure 4
Fiscal variables (% of GDP)



Source: compiled by author based on BCRP data.

The next important milestone in Peruvian economic and fiscal history came with the global financial crisis, whose impact was felt most keenly in 2008 and 2009. With regard to the variables of interest, the financial crisis precipitated a substantial downturn

After the global financial crisis, export prices reverted to an upward trend until 2013. Indeed, between 2010 and 2013, average economic growth was 6.7%; average inflation (2.8%) was located within the target range; the average primary balance was 2.4% of GDP; the public debt ratio declined, on average, by 1.9% of GDP, reaching a historic low of 19.2% of GDP in 2013; and interest payments stabilized at around 1.1% of GDP.

From 2013, the global economic context became, in general terms, unfavorable. Between 2013 and 2015, the main emerging markets were subject to considerable financial volatility, prompting an average exchange rate depreciation in excess of 10%. This context was accompanied by a continual reduction in export prices, occasioning an average economic growth rate (3.8%) some way below the average of 6.1% recorded the previous decade; or 6.5%, when the 2009 figures are excluded. Moreover, average inflation (3.2%) lay outside its target range. Compounding the high financial volatility in emerging markets, in 2017 the El Niño climatological phenomenon occurred, causing considerable infrastructural damage in the north of Peru and disruption to trade and supply chains. As a result, in 2017, economic activity slowed to a growth rate of 2.5%, while inflation (2.8%) settled just below the upper limit of the target range. Finally, in 2018, export prices and economic growth (4%) recovered, while inflation (1.3%) was firmly within its target range. However, trade disputes between the United States and China, both key trading partners of Peru, triggered new risks. This conjuncture caused the sol to depreciate by 4% as well as weakening global economic expectations, which could have a negative impact on future export prices.

With respect to the fiscal accounts, the primary balance has been in deficit continually since 2014, while public debt soared from 19.2% of GDP in 2013 to 25.8% of GDP in 2018. Consequently, interest payments went from 1.1% of GDP in 2013 to 1.4% of GDP in 2018. However, these rises may be due not only to the above-mentioned developments, but also to more effective usage of fiscal rules: a fiscal deficit of 1% of GDP, temporarily increased to fund reconstruction efforts in areas affected by El Niño; a public debt limit of 30%; and 4% of additional GDP in case of high financial volatility.

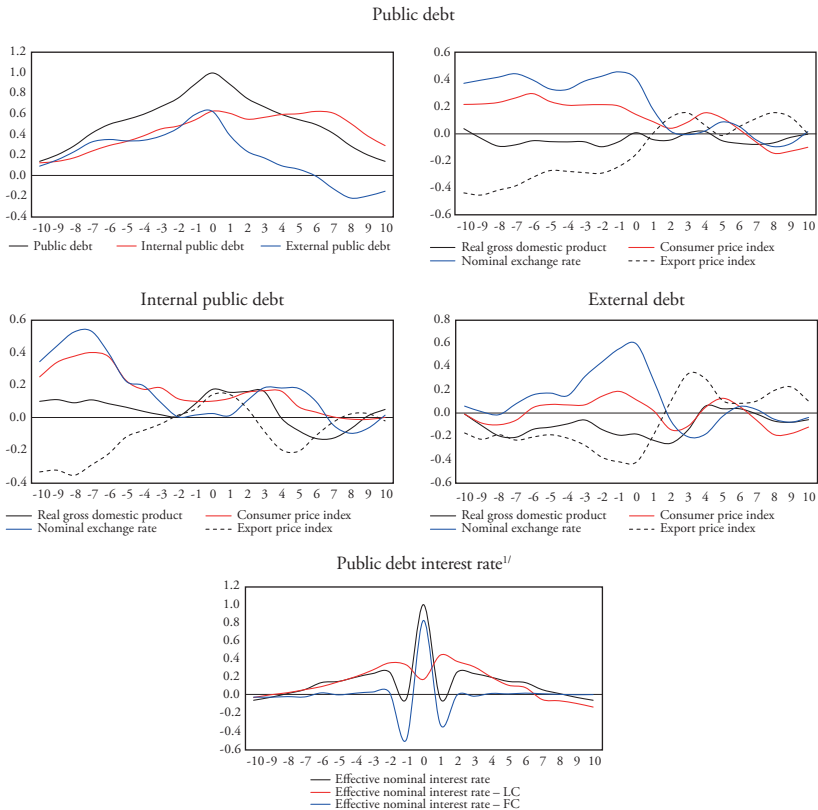
b. Cyclic cross-correlation

The analysis of cyclic cross-correlations entails the capture of real-time correlation coefficients, at current time, with lags (from $t-10$) and leads (to $t+10$) for the cyclic components¹⁸ of the relevant series.

18 To extract the cyclic component of the series studied, the Kalman (1960) filter is employed.

Figure 5 provides interesting evidence. All inferences, which are presented below, refer to cyclic components obtained for the relevant series. The first graph shows that (i) the correlation between total public debt and internal public debt leads is positive (average of 0.53 between leads t+1 and t+10), while (ii) the correlation between the total public debt and external public debt leads is decreasing to negative levels (average of -0.19 between leads t+8 and t+10). This may be due to fiscal management orientated to “solization” of public debt—that is, conversion to the Peruvian currency—to reduce exposure to exchange rate risk.

Figure 5
Cyclic cross-correlation



Note
1/ Calculations of the implicit effective nominal interest rate for the total, internal, and external interest rates (see equation 2 in Appendix 1).
Compiled by author.

In turn, the second graph shows the correlations between total public debt and the macroeconomic variables of interest. The most striking aspect is the importance of export prices and the nominal exchange rate, whose lags are linked to decreases and increases in total public debt at current time, respectively. The signs of these correlations are in line with economic theory and intuition.

The third and fourth graphs show that the correlations of internal and external public debt, respectively, with the macroeconomic variables of interest. Both graphs show similar relations to those obtained in the case of total public debt, apart from the horizon, in which the correlations are greater. Thus, for example, the greater the lags (between $t-10$ and $t-6$), the larger the correlations between internal public debt and nominal exchange rate as well as export prices; meanwhile, in the case of the external public debt, correlations with the same variables are, in absolute terms, larger when the lags are smaller, both at current time and the first lead (between $t-3$ and $t+1$). The findings regarding the exchange rate are in line with economic intuition: exchange rate depreciation affects the current external debt stock and generates incentives for future preference of internal debt. On the other hand, an exploration of the asymmetry in the correlations between export prices and types of public debt ought to be placed on the agenda for future research.

Finally, with regard to correlations between the effective nominal public debt interest rates, the rate corresponding to total public debt is most closely related with the rate for external public debt in the current period (correlation coefficient = 0.83) and with the rate for internal public debt in periods close to the current (correlation coefficient between $t-4$ and $t+4$ excluding the current period = 0.31).

Relative variables (x)	Cyclical correlation with public debt											
	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)	
Export prices	1.94	-0.19	-0.21	-0.28	-0.38	-0.42	-0.42	-0.18	0.10	0.34	0.30	0.11
				Cyclic correlation with nominal effective public-debt interest rate								
Effective nominal interest rate (ENIR)	0.15	0.20	0.24	0.26	-0.04	1.00	-0.04	0.26	0.24	0.20	0.15	
ENIR in local currency	2.49	0.15	0.20	0.36	0.33	0.17	0.44	0.37	0.31	0.20	0.11	
ENIR in foreign currency	1.36	0.00	0.02	0.03	-0.49	0.83	-0.32	0.00	-0.01	0.02	0.01	

Compiled by author.

Table 1 presents the correlation coefficients used in Figure 4, but only between $t-5$ and $t+5$. In addition, it adds to the analysis the calculation of relative volatilities, which are the standard deviation quotients of the series cycles. Thus, in the table it can be seen that the series that are more volatile than public debt are internal public debt (2.16), external public debt (1.40), and export prices (2.71). Conversely, the least volatile are real GDP (0.42), the consumer price index (0.22), and the nominal exchange rate (0.87).

Unit root tests

The first step in the statistical analysis involves evaluating the stationarity of the macroeconomic and fiscal series of interest. To this end, the Elliott-Rothenberg-Stock DF-GLS, augmented Dickey-Fuller, and Phillips-Perron unit root tests are employed. The results are presented in Table 2. The results are unanimous in finding that the series corresponding to public debt and the export price index are not stationary.

Table 2
Unit root tests

Variable	Augmented Dickey-Fuller (<i>t-stat</i>)			Phillips-Perron (adjusted <i>t-stat</i>)			Elliott-Rothenberg-Stock DF-GLS (<i>t-stat</i>)		
	None	Intercept	Intercept and trend	None	Intercept	Intercept and trend	Intercept	Intercept and trend	Intercept and trend
Export prices	0.31	-1.53	-1.74	0.60	-1.32	-1.33	-0.60	-1.86	-1.86
Export price (first difference)	-5.60**	-5.68**	-5.70**	-5.47**	-5.50**	-5.51**	-5.51**	-5.66**	-5.66**
Exchange rate variation	-2.89**	-2.86*	-3.10	-3.87**	-3.82**	-3.80**	-0.43	-1.29	-1.29
Economic growth	-1.54	-4.11**	-4.07**	-1.71*	-3.46**	-3.42*	-2.18**	-2.87*	-2.87*
Interest rate in LC	-0.82	-2.75*	-4.79**	-5.40**	-16.03**	-18.36**	-2.12**	-2.02	-2.02
Inflation	-1.00	-3.99**	-4.20**	-1.69*	-3.32**	-3.34*	-4.03**	-4.40**	-4.40**
Interest rate in FC	-0.21	-9.35**	-9.32**	0.19	-9.35**	-9.32**	-9.38**	-8.70**	-8.70**
Debt	-2.39**	-1.23	0.25	-2.22**	-1.21	0.25	-0.68	-1.19	-1.19
Debt (first difference)	-2.40**	-2.72*	-7.28**	-6.85**	-7.07**	-7.18**	-2.35**	-2.72	-2.72

Note

Significance level: 1%***, 5%** and 10%*. Compiled by authors.

3. Methodological framework

Public debt dynamics (%)

As stated above, the theoretical model corresponds to the MAC DSA framework; thus, through basic identities,¹⁹ it is possible to obtain the public debt dynamics equation.

$$d_t = \left[\frac{(1 + i_t^d)(1 - \alpha_{t-1}) + (1 + i_t^f)(1 + \varepsilon_t)\alpha_{t-1}}{(1 + \pi_t)(1 + g_t)} \right] d_{t-1} - pb_t \quad (1)$$

where d_t is the public debt stock as a percentage of GDP; i_t^d and i_t^f are the effective and implicit nominal internal and external public debt interest rates,²⁰ respectively; α_t is the proportion of public debt in foreign currency, which refers to the public debt's level of exposure to exchange rate risk; ε_t is the variation in the exchange rate; π_t is the inflation rate; g_t is the economic growth rate; and pb_t is the primary balance as a percentage of GDP.

Fiscal reaction function

Equation (1) proposes a standard analysis of the sustainability of public debt; however, this study seeks to analyze the case of an economy highly dependent on the behavior of export prices. To capture these relationships, the following fiscal reaction function (FRF) is used:

$$pb_t = \beta_0 + \beta_1 pb_{t-1} + \beta_2 d_{t-1} + \beta_3 g_{t-1} + \beta_4 px_t + \beta Z_t + \varepsilon_t \quad (2)$$

where px_t represents the export price index and Z_t refers to a vector of control variables. This specification is in line with the review of the literature associated with the FRF. This function will be estimated using various methods in order to evaluate the robustness of the results.

Autoregressive vectors model

This study is based on Celasun et al. (2006), who employ a VAR model, and on Anaya and Pienkowski (2015); although the latter employs an SVAR model, the present study borrows its usage of the lag in public debt as an exogenous variable and applies it to the VAR model to create a VAR model with an exogenous factor (VAR-X).

¹⁹ See Appendix 1.

²⁰ The interest rates, in both local and foreign currency, are quarterly annual rates.

a. Specification: VAR and VAR-X

The VAR and VAR-X models are represented by equations (3.1) and (3.2), respectively.

$$Y_t = \sum_{i=1}^p A Y_{t-i} + \mu_i \quad (3.1)$$

$$Y_t = \sum_{i=1}^p A Y_{t-i} + \sum_{i=1}^p \gamma d_{t-i} + \mu_i \quad (3.2)$$

where $Y_t = (px_t, g_t, \pi_t, \varepsilon_t, i_t^d, i_t^f)$, A_t is a coefficients matrix and μ_i represents the well-behaved error terms ($\mu_i \sim N(0, \Omega)$).

Unlike Celasun et al. (2006), this study does seek to capture possible causal relations between innovations in the relevant variables through, for example, impulse response functions (IRFs). The IRFs for public debt will be created by employing (i) the public debt dynamics equation (equation 1); (ii) the IRFs from estimating VAR and VAR-X models (equations 3.1 and 3.2); and (iii) the FRF estimated, by way of a feedback process. In line with Anaya and Pienkowski (2015), the computation of IRFs on public debt is made up of the difference between the automatic public debt trajectory and the trajectory generated by a shock on a variable of Y_t . This difference ensures that the real impact of the shock is captured. The default trajectory and that generated by a shock are subject to initial values of d_t and Y_t .²¹ Finally, it is important to note that only the statistically significant magnitudes of the IRFs obtained by estimating equations (3.1) and (3.2) will be employed in the calculation of IRFs on public debt, thereby reducing the need to estimate confidence intervals.

b. Granger causality tests

Given that IRFs are sensitive to the order of the variables, it is necessary to establish *ex ante* a causal relationship. To this end, Table 3 shows the results obtained from the Granger causality tests.

21 The initial values of d_t , i_t^d , i_t^f and α_t are their respective values at the close of 2018. For their part, the initial values of g_t , π_t and ε_t are their respective long-term values. Finally, the initial value of px_t corresponds to its respective value in its base year. The calculations of the IRFs on public debt are not sensitive to marginal changes in the initial values.

Table 3
Granger causality tests

Null hypothesis			Prob.	Null hypothesis			Prob.	Null hypothesis			Prob.
px	↔	if	0.19	id	↔	px	0.69	π	↔	px	0.52
px	↔	id	0.01	id	↔	if	0.15	π	↔	if	0.97
px	↔	g	0.04	id	↔	g	0.45	π	↔	id	0.04
px	↔	π	0.34	id	↔	π	0.02	π	↔	g	0.02
px	↔	ε	0.00	id	↔	ε	0.65	π	↔	ε	0.69
if	↔	px	0.49	g	↔	px	0.65	ε	↔	px	0.94
if	↔	idx	0.68	g	↔	if	0.47	ε	↔	if	0.54
if	↔	g	0.22	g	↔	id	0.02	ε	↔	id	0.49
if	↔	π	0.97	g	↔	π	0.32	ε	↔	g	0.04
if	↔	ε	0.69	g	↔	ε	0.35	ε	↔	π	0.13

Compiled by authors.

The results show that the most suitable ordering of the variables is: $(px_t, \varepsilon_t, g_t, i_t^d, \pi_t, i_t^f)$. Taking into these account these results, as well as those of Anaya and Pienkowski (2015), restrictions are placed upon the coefficient matrix of the VAR and VAR-X models, such that: (i) the export price index can only be affected by innovations in themselves; (ii) domestic and external interest rates can only affect each other; and (iii) innovations on inflation do not have an impact on economic growth.

c. Optimal lag

Before performing the estimation and analyzing the results, it is necessary to determine the optimal number of lags that the VAR and VAR-X should take into account. To this end, Table 4 shows the optimal lag for each model based on different statistical tests and the number of lags used.

Table 4
Determination of optimal lag

Number of lags analyzed	VAR model					VAR-X model				
	LR	FPE	AIC	SC	HQ	LR	FPE	AIC	SC	HQ
1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	1	2	2	2	2	1	2
3	2	2	2	1	2	2	2	2	1	2
4	2	2	2	1	2	2	2	2	1	2
5	2	2	2	1	2	2	2	2	1	1
6	2	2	2	1	2	2	2	2	1	1
7	2	2	7	1	2	2	2	7	1	1
8	8	2	8	1	2	8	2	8	1	1
9	8	9	9	1	9	2	8	8	1	9

Notes

LR: Statistic from modified sequential likelihood ratio test.

FPE: Final prediction error.

AIC: Akaike information criterion.

SC: Schwarz information criterion.

HQ: Hannan-Quin information criterion.

Compiled by author.

Table 4 shows that the best recommendation is two lags in each model. For the estimation of both models, the restrictions on the coefficient matrix will be applied for both lags.

4. Results

Fiscal reaction function

Table 5 presents the estimation of the FRF by way of five different statistical techniques. The first column shows the results of an estimation that uses the ordinary least squares (OLS) model, which is the least preferable due to the assumptions it entails. The second column displays the results of a two-stage least squares (TSLS) model, which is useful because the lags generate feedback in the FRF, and given the possible error correlation in the dependent variable with the explanatory variables. The third and four columns show results of the FRF estimated using the Gaussian mixture model (GMM), which considers the use of instruments to address possible endogeneity. The difference between the two columns lies in the way that possible heteroskedasticity is addressed: the third column uses White's method, while the fourth employs a heteroskedasticity- and autocorrelation-consistent (HAC) weighting matrix. Finally, the fifth columns exhibit

the results of the limited-information maximum-likelihood (LIML) model, which will be preferable to the GMM if the instruments employed are not particularly strong.

All estimations are mutually consistent, at least in terms of significance and sign, which attests to the robustness of the results. For some of the variables, the magnitudes of the estimations under the OLS model differ significantly from the estimations under the other models, which alerts us to the possible presence of problems of endogeneity and/or heteroskedasticity. All estimations suggest that the primary balance has a positive response to public debt, as a percentage of GDP in the case of both the primary balance and public debt. Moreover, the positive and significant reaction of the primary balance to economic growth suggests that this growth is countercyclical. Finally, all measures point to the primary balance reacting strongly to the export price index. Thus, all results are in line with economic theory and intuition.

Table 5
FRF estimation

Dependent variable: primary balance (% of GDP)	OLS	TOLS	GMM White	GMM HAC	LIML
Primary result (% of GDP, t-1)	0.35**	0.52**	0.56**	0.56**	0.45**
Public debt (% of GDP, t-1)	0.13**	0.10*	0.09*	0.08**	0.10*
Economic growth (% , t-1)	0.17**	0.18**	0.15**	0.17**	0.19**
Export price index	4.80**	3.43*	3.00*	2.70**	3.75*
Constant	-8.75**	-7.28**	-6.96**	-6.27**	-7.47**
R2	0.829	0.834	0.828	0.830	0.838
Adjusted R2	0.809	0.813	0.808	0.809	0.818

Notes

Significance level: 1%***, 5%** and 10%*.

In each of the regressions, the only variables instrumented are the first lag of the primary balance and the first lag of public debt. For the TOLS model, the second lags of public debt, the economic growth rate, the export price index, the interest rate in foreign currency, and the annual exchange rate variation are utilized as instruments. In the case of the GMM models, the instruments are: quarterly dummies, the second lag in the primary balance, the second and third lags of public debt, the second and third lags in economic growth, the first and second lags in the export price index, the interest rates in local and foreign currency, and annual exchange rate variation. The LIML model employ the second and fourth lags of public debt, the second and third lags of economic growth and the export price index, the second lag of the interest rate in foreign currency, and the annual exchange rate variation.

Unreported controls (see Appendix 2).

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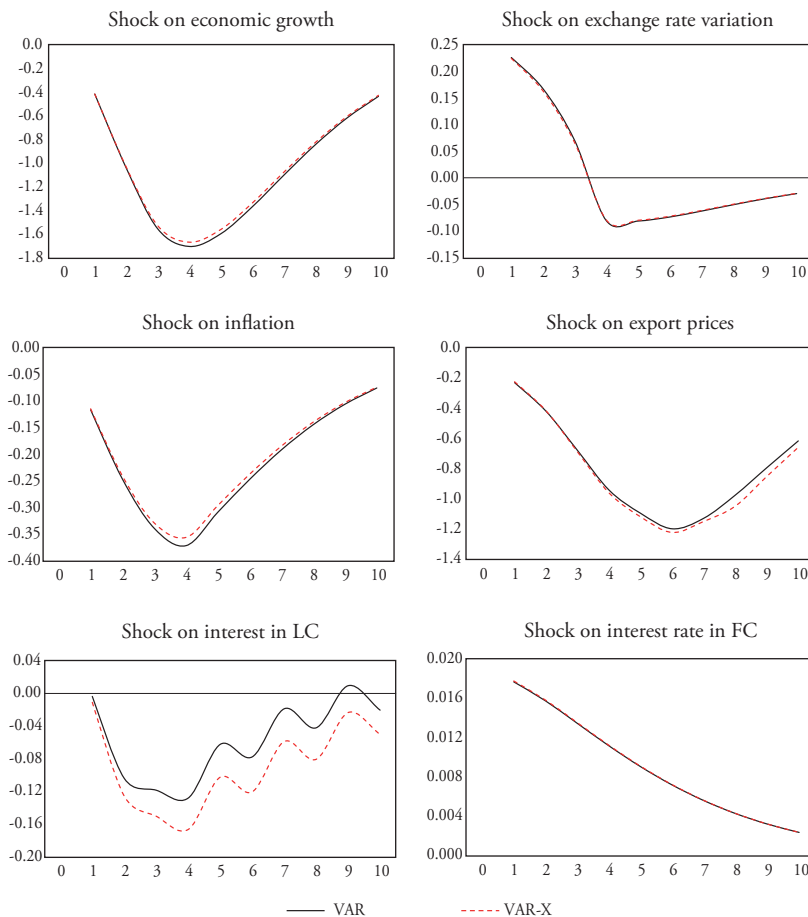
For calculation of the IRFs on public debt, the FRFs estimated by way of the GMM with HAC correction are employed, since their results demonstrate a higher level of significance for the explanatory variables. However, the

IRFs on public debt are also calculated using the FRFs estimated under the other methods. These results, which demonstrate robustness, are presented in appendices 3 and 4.

Impulse response function

Figure 6 demonstrates the behavior of public debt in response to shocks on the variables of interest. First, it can be seen that the inclusion of public debt lagged as an exogenous variable in the VAR-X model has not given rise to results that are significantly different from those obtained under the VAR model. Thus, the results are robust to both types of specifications.

Figure 6
IRF on public debt (% of GDP)



Source: Compiled by author.

Specifically, an improvement in the real economic growth rate reduces public borrowing needs in the short, medium, and long term. The main impact occurs in the fourth quarter, with a reduction in public debt equivalent to 1.67% of GDP. Likewise, a higher rate of inflation will also reduce the public debt ratio, since it directly affects nominal GDP. The greatest impact occurs in the fourth quarter (-0.37% of GDP). Moreover, a shock on export prices also negatively affects public debt, but unlike economic growth and inflation its effects are greater in the long term. Thus, the greatest debt reaction to a shock on export prices occurs in the sixth quarter (-1.22% of GDP).

Conversely, a depreciation shock on the exchange rate brings about an increase in public debt in the short term, of approximately 0.23% of GDP. However, this effect fades rapidly, and from the fourth quarter there is a reduction in the rate of public borrowing, which may be due to the decreasing exposure of public debt to exchange rate risk and/or a possible reaction in fiscal management.

When it comes to interest-rate innovations, the opposite effects are observed. On the one hand, a shock on the nominal interest rate in local currency has a slight negative impact on public debt. The reason is that although the interest rate is associated with a higher cost of public financing, it is also associated positively with the inflation rate²²—and it is this which offsets and overcomes the effects of the shock on the interest rate in local currency, thereby reducing public debt. On the one hand, shocks on the nominal interest rate in foreign currency have a slight negative impact on public debt. However, these impacts are small, possibly because 90% of the public debt is fixed rate, which results in very little exposure to financial volatility.

In sum, all results are consistent with financial intuition and with the analytical framework applied. Moreover, it was found that the results are robust to the employment of lagged public debt as an exogenous variable in the VAR modeling. For this reason, the next section develops projections based solely on the VAR model (equation 3.1).

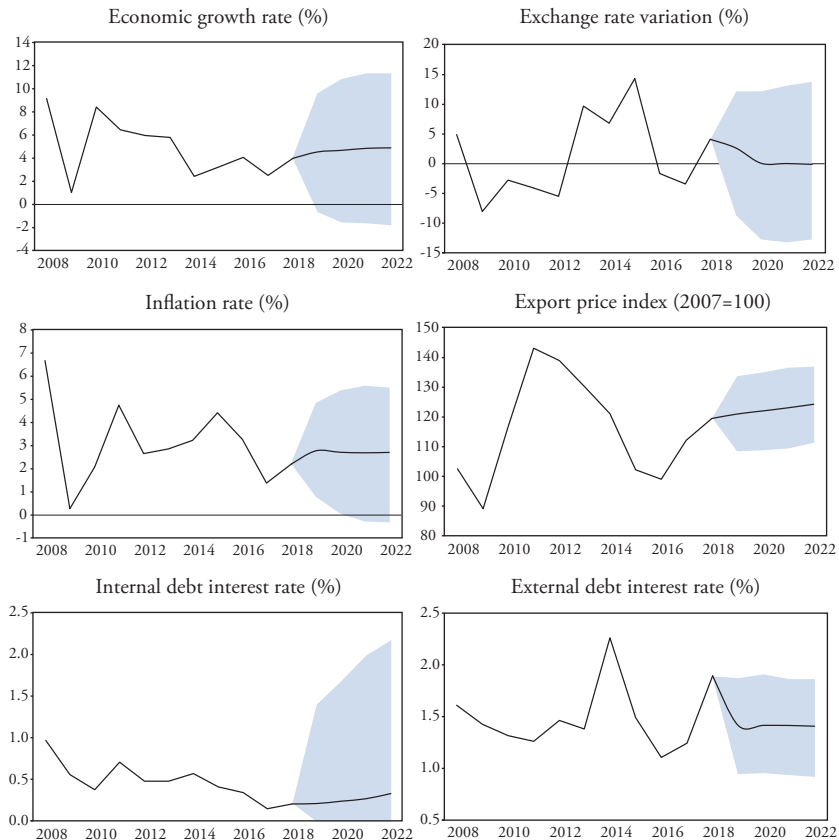
Projections

Finally, the predictive power of the VAR model is utilized to map out future public debt trajectories and gain a better perspective of the Peruvian economy's present conjuncture in terms of public debt sustainability. These

22 See the results of the Granger causality tests for the inflation rate and the nominal interest rate in local currency.

trajectories correspond to (i) a base scenario obtained from the VAR model itself; and (ii) paths that allow the model's results to be compared with official public debt projections presented in the 2020-2023 Multiannual Macroeconomic Framework (MMF). Moreover, the projections correspond to a four-year time horizon; that is, until the fourth quarter of 2022.

Figure 7
Projection of the determinants of public debt dynamics



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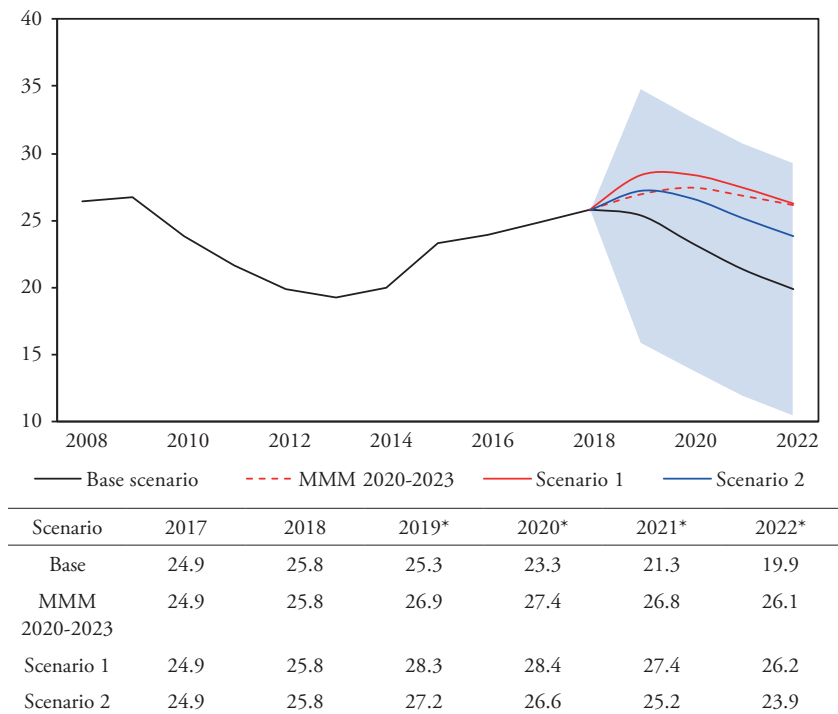
As to the base scenario, Figure 7 shows the projections of the public debt determinants, obtained from the VAR model. Thus, the base scenario implies an average growth rate of 4.8% for the next four years. Moreover, the depreciation pressures on the local currency are expected to gradually disappear. The average inflation rate is assumed to be 2.6%. In turn, the export price index gradually increases until it reaches 124.5 points in 2022.

When it comes to interest rates, the rate in local currency increases slightly, from 0.21% in 2018 to 0.34% in 2022, while the rate in foreign currency stabilizes at 1.86%.

Consequently, Figure 8 shows various future trajectories for public debt. On the one hand, the most optimistic projection is that of the base scenario, which marks a gradual reduction in the public debt ratio to 19.9% of GDP in 2022. This scenario reproduces the debt ratios witnessed over the 2012–2014 period. It is important to note that this trajectory depends on the following projection assumptions regarding the determinants of debt dynamics: brisk economic activity; a controlled rate of inflation; improvements to export prices; low volatility in interest rates; and a context of little to no depreciation pressure on the local currency. However, these projections differ from current expectations about the determinants of public debt. Thus, additional paths are established to enable analysis of public debt sustainability based on more restrictive assumptions.

On the one hand, the dotted line in Figure 8 corresponds to the current projection of public debt presented in the MMM. Scenario 1 corresponds to the application of MMM assumptions only in the equation of public debt dynamics, without taking into account the interrelations of the VAR model. It should be noted that Scenario 1 takes into account a higher debt-to-GDP ratio in the initial years of the projection, but then coincides up to 2022. Because the VAR model interrelations are not taken into account, the difference between Scenario 1 and the MMM projection are primarily due to the role of the FRF estimated.

Figure 8
Projection of public debt



Compiled by author.

Thereafter, Scenario 2 also employs the MMM assumptions for public debt determinants, but unlike Scenario 1, it does take into account the interrelations estimated under the VAR model. Thus, Scenario 2 projections are close to those of the MMM in the first two years of projection, but then decrease and diverge. This may be because the MMM projections do not take into account the interrelations between the determinants of public debt dynamics.²³

5. Conclusions

This article analyzes the sustainability of public debt, defined as the dynamics of the debt-to-GDP ratio, in an economy rich in natural resources with access to international capital markets. Under the MAC DSA with a focus

²³ The MMM employs the methodological strategy developed by Celasun et al., but only for preparing fan charts, while the only projection path is developed deterministically.

on export-price dependency, the factors that affect public debt sustainability are expected to be economic growth, exchange rate depreciation, inflation, the cost of public financing in local and foreign currency, and export prices.

To this end, VAR and VAR-X models in macrofiscal statistics from the Peruvian economy for the period 1999-2018 were employed. The modeling of this type of models fulfills the need to capture interrelations between the abovementioned macrofiscal variables and incorporate them into the public debt dynamics equation. Moreover, the VAR-X specification uses the first lag in the debt-to-GDP ratio as an exogenous variable.

Thus, impulse-response functions were constructed to examine the reaction of public debt to innovations in economic growth, currency depreciation, inflation, the nominal exchange rate in both local and foreign currency, and export prices. The results attest to robustness between both specifications of the autoregressive models. Moreover, the findings for the short and medium term were consistent with economic theory. Positive shocks to currency depreciation and the interest rate in foreign currency cause increases in the debt ratio, while positive shocks to economic growth, inflation, and export prices prompt a reduction in this ratio. In turn, long-term shocks occur only in the case of shocks to export prices, due to their positive impact on tax collection. Finally, innovations in the interest rate in local currency have negative impacts on public debt in the short term, given the strong relationship with inflation and because the effect is limited by the high proportion of public debt at fixed rate. Consequently, different scenarios for the public debt ratio are proposed. The base scenario allows for a debt ratio of 19.9% of GDP by the close of 2022 if the economy returns to a growth rate close to 5%, accompanied by a gradual reduction in depreciation pressure on the sol, control of the inflation rate, and a slow but progressive improvement of export prices. In addition, the assumptions of the 2020-2023 MMF were employed to develop scenarios that enable analysis of fiscal sustainability under more restrictive macroeconomic conditions. The paths obtained are relatively close to the MMF projections, which highlights the validity of the FRF estimations and the interrelations obtained from the VAR model. Moreover, the paths underscore the importance of creating favorable macroeconomic contexts to assure fiscal sustainability, taking into account, in particular, the limited current room for maneuver given the fiscal rule of 30% of GDP.

All of the above verifies the importance of economic growth, inflation, currency depreciation, public borrowing costs, and export prices on the sustainability of public debt. This shows that opportunities and risks remain for macrofiscal policy. Thus, future measures of fiscal sustainability should: (i)

continue with measures to “solize” public debt and increase the proportion of fixed-rate interest, which involves keeping the current public borrowing profile; and (ii) diversify sources of fiscal income in order to reduce the vulnerability of fiscal sustainability to unfavorable changes in export prices. These recommendations will reinforce low levels of public borrowing and sound fiscal management, both key indicators for the major rating agencies.

References

- Adedeji, O., & Williams, O. (2007). *Fiscal reaction functions in the CFA Zone: An analytical perspective*. Working Paper 7-232. Washington, D.C.: International Monetary Fund.
- Alfonso, A. (2005). Fiscal sustainability: The unpleasant European case. *FinanzArchiv: Public Finance Analysis*, 61(1), 19-44.
- Afonso, A., & Jalles, J. (2011). *Appraising fiscal reaction functions*. ISEG-UTL Economics Working Paper 23.
- Anaya, P., & Pienkowski, A. (2015). *What really drives public debt: A holistic approach*. Working Paper 15-137. Washington, D.C.: International Monetary Fund.
- Arango, L., & Posada, C. (2001). ¿Podremos sostener la deuda pública? Borradores de Economía 165. Bogota: Banco de la República.
- Bachellerie, A., & Couillault, B. (2005). Public debt sustainability and crises in emerging market countries: A presentation of the concepts and diagnostic tools. *Financial Stability Review*, 6, 63-80. Paris: Banque de France.
- Banco Central de Reserva del Perú (BCRP). (n.d.). Official website. <http://www.bcrp.gob.pe>
- World Bank & International Monetary Fund. (2012). *Revisiting the debt sustainability framework for low-income countries*. Washington, D.C.
- World Bank. (2005). *How to do a debt sustainability analysis for low income countries*. Washington, D.C. World Bank. (n.d.). Official website. <http://www.worldbank.org>
- Burger, P., & Marinkov, M. (2012). Fiscal rules and regime-dependent fiscal reaction functions. *OECD Journal on Budgeting*, 12(1), 1-29.
- Celasun, O., Debrum, X., & Ostry, J. (2006). Primary surplus behavior and risks to fiscal sustainability in emerging market countries: A “fan-chart” approach. *IMF Staff Papers*, 53(3), 401-425. Washington, D.C.: International Monetary Fund (IMF).
- Checherita-Westphal, C., & Ždarek, V. (2015). Fiscal reaction function and fiscal fatigue in the euro area. In *Public Finance Workshop*, Hrvatska Narodna Banka, Zagreb.
- Córdova, J. & Rojas, Y. (2010). Reglas fiscales y términos de intercambio. *Revista Estudios Económicos*, 19, 7-32. Lima: Banco Central de Reserva del Perú (BCRP).
- De Mello, L. (2008). Estimating a fiscal reaction function: The case of debt sustainability in Brazil. *Applied Economics*, 40(3), 271-284.
- Díaz, C., & Gutiérrez, F. (2005). *Deuda implícita y contingencias estatales: un análisis de sostenibilidad para el caso colombiano*. Bogota: Pontificia Universidad Javeriana.
- Dobbs, R., Lund, S., Woetzel, J., & Mutafchieva, M. (2015). *Debt and (not much) deleveraging*. McKinsey Global Institute.
- Dore, O., & Masson, P. (2002). *Experience with budgetary convergence in the Waemu*. Working Paper 2-108. Washington, D.C.: International Monetary Fund.
- El Mahmah, A., & Kandil, M. (2019). Fiscal sustainability challenges in the new normal of low oil prices. *International Journal of Development Issues*, 18(1), 109-134.
- International Monetary Fund (IMF). (2002). *Assessing sustainability*. Washington, D.C.
- International Monetary Fund (IMF). (n.d.). Official website. <http://www.imf.org>
- Galvis, D., Quintero, B., & Rhenals, R. (2008). Dinámica y sostenibilidad de la deuda pública colombiana: ¿existe margen para una política fiscal contracíclica? *Perfil de Coyuntura Económica*, 12, 85-116. Medellín: Universidad de Antioquia.

- Galvis, J. (2015). Evaluación empírica de una función de reacción fiscal para Colombia, 2000-2012. *Lecturas de Economía*, (82), 57-92.
- Ganiko, G., Melgarejo, K., & Montoro, C. (2016). *Estimación del espacio fiscal en economías emergentes: el caso peruano*. Research paper 001-2016. Peru: Consejo Fiscal del Perú.
- Ganiko, G., & Montoro, C. (2017). *Reglas fiscales para exportadores de commodities: una aplicación para Perú*. Research paper 001-2017. Peru: Consejo Fiscal del Perú.
- Golinelli, R., & Momigliano, S. (2007). *The cyclical response of fiscal policies in the euro area—why do results of empirical research differ so strongly?* Economic Working Papers 654. Bank of Italy.
- Herrera, P., & Mendoza, W. (2004). *La sostenibilidad de la deuda pública en una economía abierta*. Working paper 230. Peru: Pontificia Universidad Católica del Perú.
- Kalman, R. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82, 35-45. United States: Transactions of the ASME.
- Lanteri, L. (2015). Resultado fiscal, cuenta corriente y términos de intercambio. Relaciones de largo plazo para la economía argentina. *Revista Análisis Económico*, XXX(74), 97-113. Mexico: Universidad Autónoma Metropolitana.
- Luporini, V. (2000). Sustainability of the Brazilian fiscal policy and central bank independence. *Revista Brasileira de Economia*, 54(2), 201-226.
- McKinsey & Company. (2015). *Debt and (not much) deleveraging*. United States: McKinsey & Company.
- Ministerio de Economía y Finanzas (MEF). (2019). *Marco macroeconómico multianual 2020-2023*. Perú.
- Ministerio de Economía y Finanzas (MEF). (n.d.). Official website. <http://www.mef.gob.pe>
- Ncube, M., & Brixiová, Z. (2015). *Public debt sustainability in Africa: Building resilience and changes ahead*. Working Paper Series 227. Côte d'Ivoire: African Development Bank Group.
- Pastén, R. (2007). A note on fiscal deficit sustainability in Chile: 1833-1999. *Panorama Económico*, 25(35), 160-167.
- Pereyra, J. (2003). *Sostenibilidad de la política fiscal: una simulación de la restricción presupuestaria*. Series de Documentos de Trabajo 2003-009. Peru: Banco Central de Reserva del Perú (BCRP).
- Plödt, M., & Reicher, C. (2015). Estimating fiscal policy reaction functions: The role of model specification. *Journal of Macroeconomics*, 46, 113-128.
- Qin, D., Cagas, M., Ducanes, G., Magtibay-Ramos, N., & Quising, F. (2005). *Empirical assessment of sustainability and feasibility of government debt: The Philippines case*. ERD Working Papers Series 64. Philippines: Asian Development Bank.
- Tanner, E., & Samake, I. (2008). Probabilistic sustainability of public debt: A vector auto-regression approach for Brazil, Mexico, and Turkey. *IMF Staff Papers*, 55(1), 149-182.
- U.S. Geological Survey. (2019). *Mineral commodity summaries 2019*. <https://doi.org/10.3133/70202434>.

Appendices

Appendix 1

Debt Sustainability Analysis Framework for Market-Access Countries (MAC DSA)²⁴

The theoretical model employed in this article corresponds to the MAC DSA, with a focus on dependence on the terms of trade. First, the debt stock dynamics are expressed as follows:

$$D_t = D_{t-1} + \Delta D_t \quad \dots (1)$$

Where D_t represents the debt stock in period t . On the other hand, expenditure on interest or on debt service is represented as follows:

$$i_t = i_t^w, D_{t-1} \quad \dots (2)$$

where i_t^w represents the nominal interest rate. Likewise, the primary balance PB_t is calculated as the difference between total income R_t and total non-financial income:

$$PB_t = R_t - G_t \quad \dots (3)$$

Moreover, the nominal interest rate can be broken down into the real interest rate r_t and the inflation rate π_t , following the Fisher equation:

$$(1 + i_t^w) = (1 + r_t)(1 + \pi_t) \quad \dots (4)$$

Finally, nominal GDP can be expressed as its past value multiplied by its respective growth and inflation rates:

$$P_t Y_t = (1 + \pi_t) P_{t-1} (1 + g_t) Y_{t-1} \quad \dots (5)$$

where P_t is the consumer price index and Y_t is real GDP. Moreover, g_t is the economic growth rate.

Now, it should be recalled that financing needs are positively related to the fiscal deficit, as well as to other external transactions OT_t (for example, debt forgiveness or donations to other countries) and to the debt increase due to interest. Therefore:

$$\Delta D_t = G_t + i_t^w D_{t-1} - R_t + OT_t \quad \dots (6)$$

²⁴ IMF and World Bank (2012).

By replacing (6) in (1):

$$G_t + i_t^w D_{t-1} - R_t + OT_t = D_t - D_{t-1} \quad \dots(7)$$

$$i_t^w D_{t-1} - PB_t + OT_t = D_t - D_{t-1} \quad \dots(8)$$

On the other hand, it is possible to break down public debt stock into the base currency:

$$D_t = D_t^d + e_t D_t^f \quad \dots (9)$$

Where: D_t^d is the local currency, D_t^f is the foreign currency, and e_t is the nominal exchange rate. By incorporating (8) in (9), we obtain:

$$D_t^d + e_t D_t^f = (1 + i_t^d) D_{t-1}^d + (1 + i_t^f) e_t D_{t-1}^f - PB_t + OT_t \quad \dots(10)$$

On this point, the second element of the right member of (10) is multiplied by e_{t-1} and D_{t-1} , giving:

$$(1 + i_t^f) e_t D_{t-1}^f \cdot \frac{1}{e_{t-1}} \cdot e_{t-1} \cdot \frac{1}{D_{t-1}} \cdot D_{t-1} \quad \dots (11)$$

To reorder:

$$(1 + i_t^f) \frac{e_t}{e_{t-1}} \cdot \frac{e_{t-1} D_{t-1}^f}{D_{t-1}} \cdot D_{t-1} \quad \dots (12)$$

In this way, α_{t-1} can be defined as the share of foreign debt as part of total debt in period $t - 1$; $(1 - \alpha_{t-1})$ as the share of domestic debt as part of total debt in period $t - 1$; and $(1 + \varepsilon_t)$ as exchange rate depreciation:

$$(1 + i_t^f)(1 + \varepsilon_t) \alpha_{t-1} D_{t-1} \quad \dots (13)$$

Thus (10) can be rewritten as:

$$D_t = (1 + i_t^d)(1 - \alpha_{t-1}) D_{t-1} + (1 + i_t^f)(1 + \varepsilon_t) \alpha_{t-1} D_{t-1} - PB_t + OT_t \quad \dots(14)$$

Now, (14) will be divided by nominal GDP (5). Each quotient is expressed in lower case to denote it as a fraction of GDP:

$$\frac{D_t}{P_t Y_t} = \frac{(1 + i_t^d)(1 - \alpha_{t-1}) D_{t-1}}{(1 + \pi_t)(1 + g_t) P_{t-1} Y_{t-1}} + \frac{(1 + i_t^f)(1 + \varepsilon_t) \alpha_{t-1} D_{t-1}}{(1 + \pi_t)(1 + g_t) P_{t-1} Y_{t-1}} - \frac{PB_t}{P_t Y_t} + \frac{OT_t}{P_t Y_t} \quad \dots(15)$$

$$d_t = \left[\frac{(1 + i_t^d)(1 - \alpha_{t-1}) + (1 + i_t^f)(1 + \varepsilon_t) \alpha_{t-1}}{(1 + \pi_t)(1 + g_t)} \right] d_{t-1} - pb_t + ot_t \quad \dots(16)$$

In sections 4 and 5, ot_t is excluded from the methodological strategy, because: (i) it is not significant a tlevels, or as a percentage of GDP; and

(ii) it is composed of very specific accounts, about which too little data is available. Thus, ot_t is not relevant for analysis of the Peruvian case, though it may be so for another economy.

Appendix 2
FRF estimation

Dependent variable: primary balance (% of GDP)	OLS	TOLS	GMM White	GMM HAC	LIML
Primary result (% of GDP, t-1)	0.35**	0.52**	0.56**	0.56**	0.45**
Public debt (% of GDP, t-1)	0.13**	0.10*	0.09*	0.08**	0.10*
Economic growth (% , t-1)	0.17**	0.18**	0.15**	0.17**	0.19**
Export price index	4.80**	3.43*	3.00*	2.70**	3.75*
Fiscal advice dummy	-1.55**	-0.98	-0.63	-0.89**	-1.19
One-off dummy	6.20**	6.13**	6.40**	6.27**	6.26**
First-quarter dummy	6.35**	7.69**	8.38**	8.12**	7.11**
Fourth-quarter dummy	-4.14**	-3.78**	-3.66**	-3.37**	-4.04**
Constant	-8.75**	-7.28**	-6.96**	-6.27**	-7.47**
R2	0.829	0.834	0.828	0.830	0.838
Adjusted R2	0.809	0.813	0.808	0.809	0.818

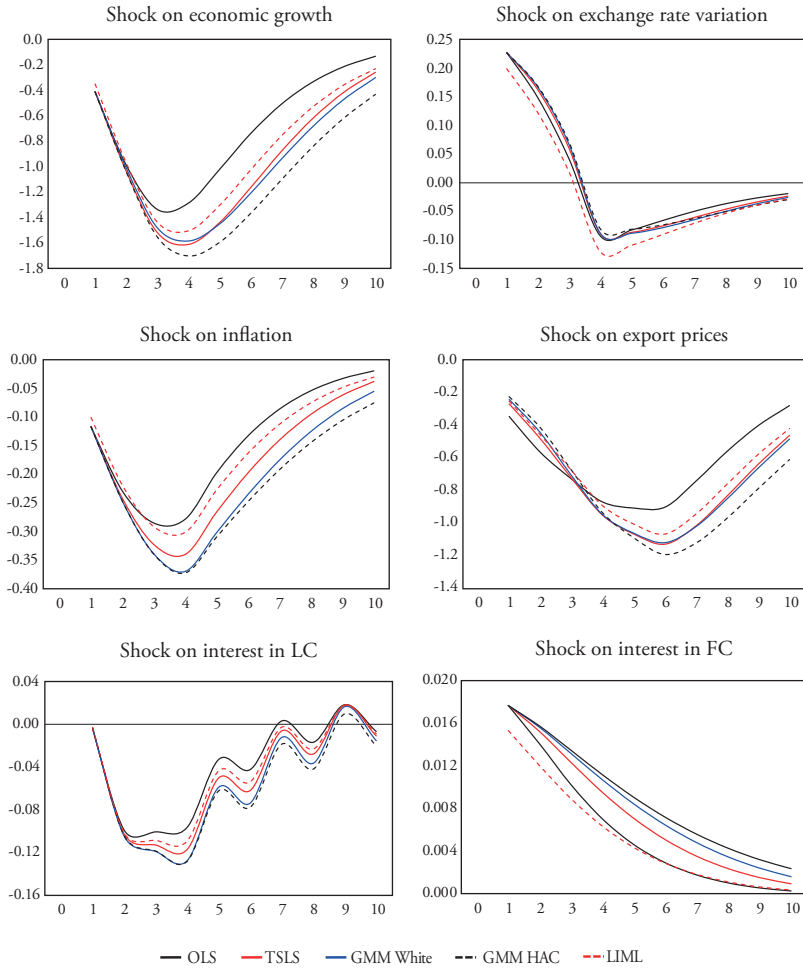
Notes

Significance level: 1%***, 5%** and 10%*.

In each of the regressions, the only variables instrumented are the first lag of the primary balance and the first lag of public debt. For the TOLS model, the second lags of public debt, the economic growth rate, the export price index, the interest rate in foreign currency, and annual exchange rate variation are utilized as instruments. In the case of the GMM models, the instruments are: quarterly dummies, the second lag of the primary balance, the second and third lags of public debt, the second and third lags in economic growth, the first and second lags in the export price index, the interest rates in local and foreign currency, and annual exchange rate variation. The LIML model employ the second and fourth lags of public debt, the second and third lags of economic growth and the export price index, the second lag of the interest rate in foreign currency, and annual exchange rate variation.

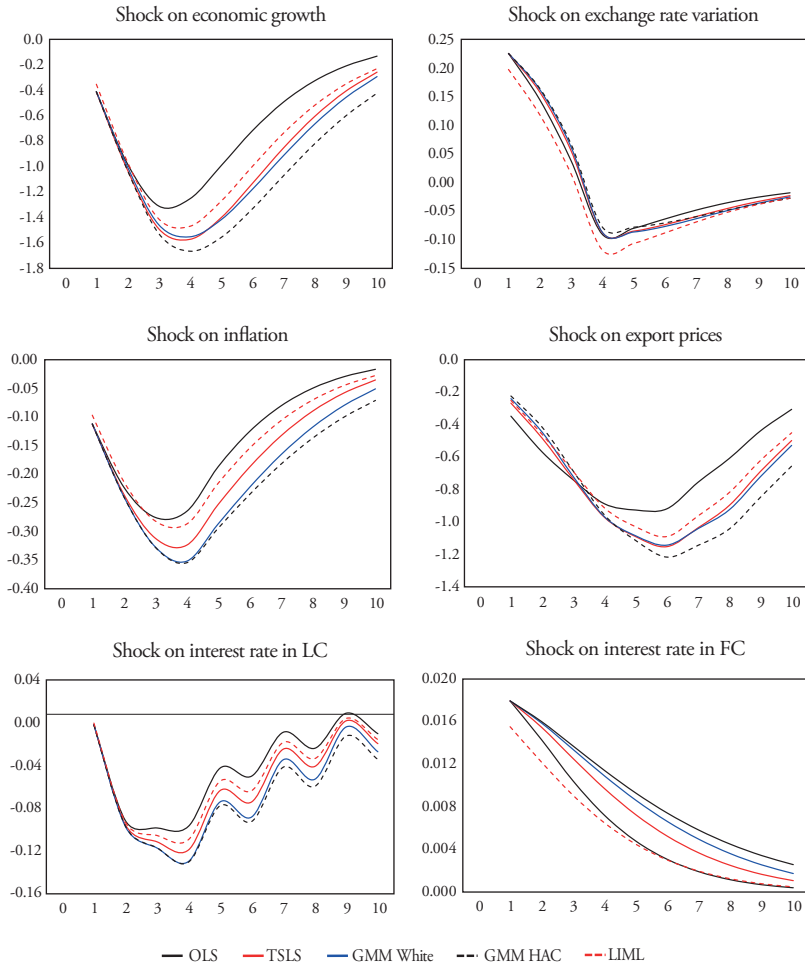
Compiled by author.

Appendix 3
 IRF on public debt (% of GDP) – VAR model



Source: Compiled by author.

Appendix 4
 IRF on public debt (% of GDP) – VAR-X model



Source: Compiled by author.

